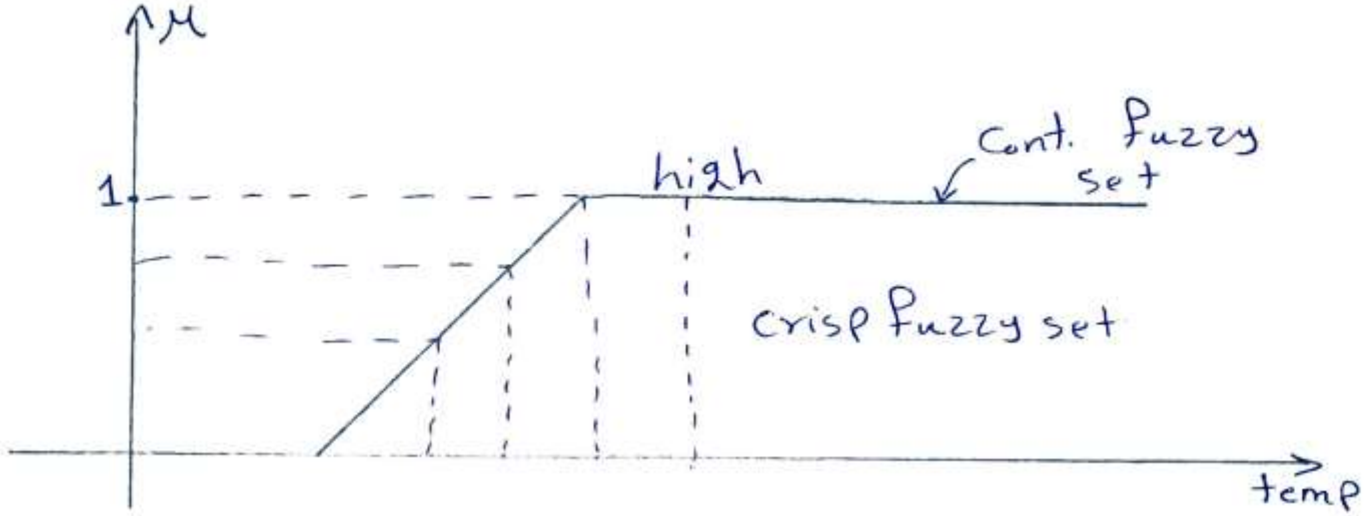
Fuzzy control

Lec 2

Fuzzy sets:

- → Crisp fuzzy set (discrete)
- → Cont. fuzzy sets



* The crisp fuzzy sets:

→ For any crisp fuzzy sets : A "label for fuzzy set" we can write the mathematical form to represent "A" as :

1) $A = \{(x_i, \mu_A(x_i)) \mid x_i \in X\}$

$; i = 1, 2, \ldots n$

where :

- x_i → the elements of the crisp fuzzy sets or the member of the " " " .
- n → no. of the elements in the crisp fuzzy set.

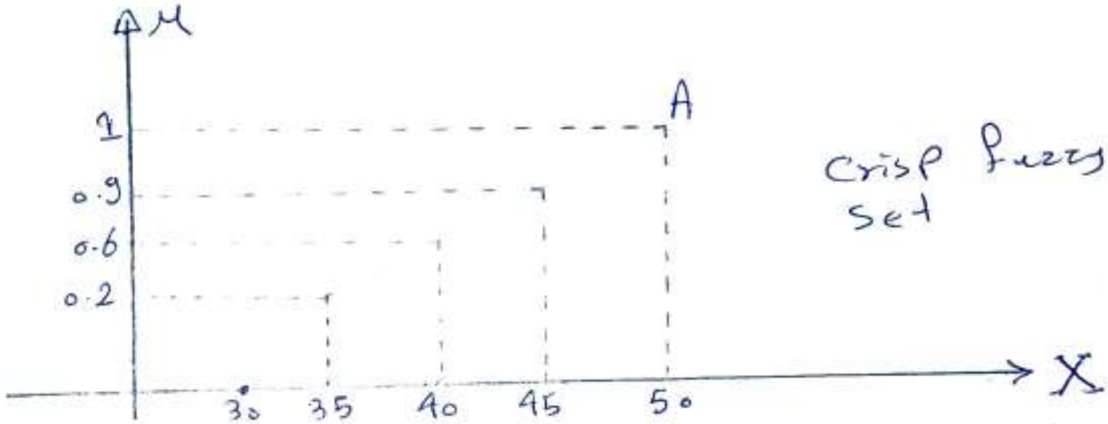
X: is the universe of discourse.

جميع القيم المتاحة لل (fuzzy set) التي من الممكن الاختيار منها.

* Universe of discourse

↳ range of all possible values considered to fuzzy sets. (القيم المتاحة على المحور الأفقي)

- $\mu_A(x_i) \rightarrow$ degree or grade of the belonging x_i to the fuzzy set A $(0 \rightarrow 1)$



* Method (1)

$$A = \{(30, 0), (35, 0.2), (40, 0.6), (45, 0.9), (50, 1)\}$$

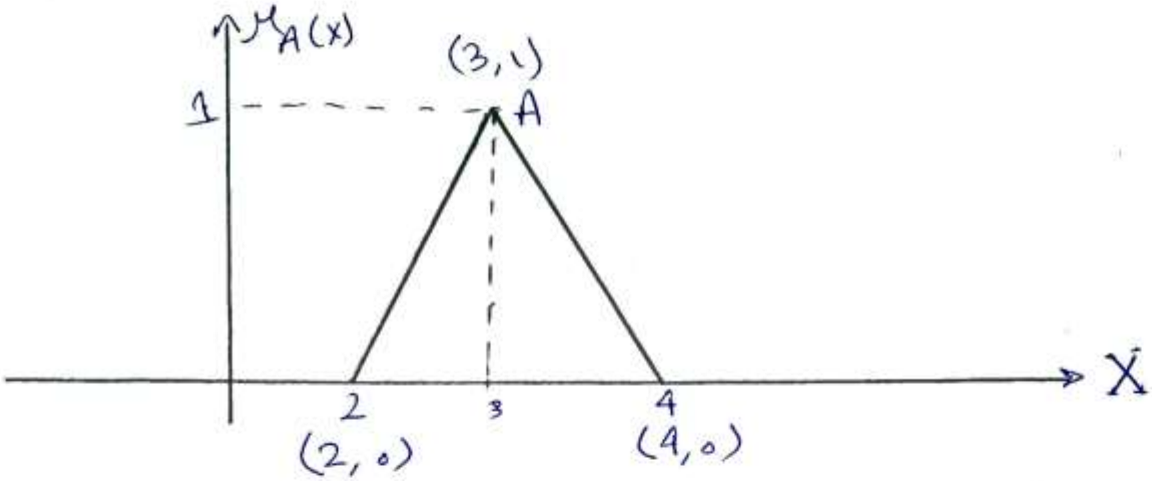
element ↲ ↳ μ

* Method (2)

$$\left\{ \sum_{i=1}^{n} \mu_A(x_i) / x_i \;\middle|\; x_i \in X \right\}$$

$$A = \left\{ \frac{0}{30}, \frac{0.2}{35}, \frac{0.6}{40}, \frac{0.9}{45}, \frac{1}{50} \right\}$$

note: method 1 & 2 are being used to describe fuzzy sets.



⇒ to describe cont. fuzzy sets

1) graphical (previous figure)

2) $\mu_A(x) = \begin{cases} \ldots\ldots & 2 \le x \le 3 \\ \ldots\ldots & 3 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$

↳ $\mu_A = \begin{cases} x-2 & 2 \le x \le 3 \\ -x+4 & 3 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$

→ $\dfrac{\mu_A - 0}{x-2} = \dfrac{1-0}{3-2} \Rightarrow \mu_A = x-2$

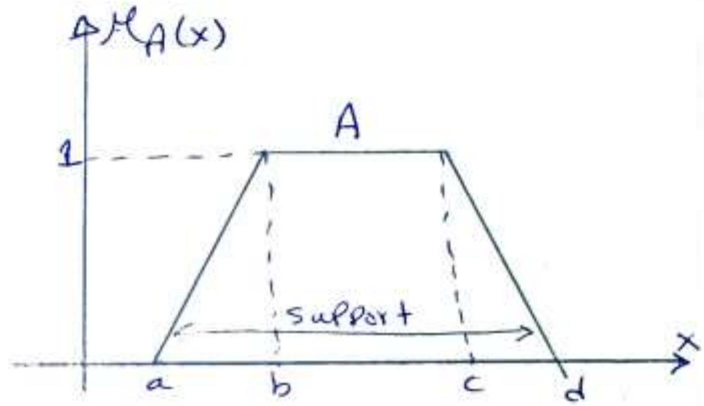
→ $\dfrac{\mu_A - 0}{x-4} = \dfrac{1-0}{3-4} \Rightarrow \mu_A = -x+4$

* The most common type is the cont. fuzzy sets:-

⇒ other concepts about fuzzy sets:.

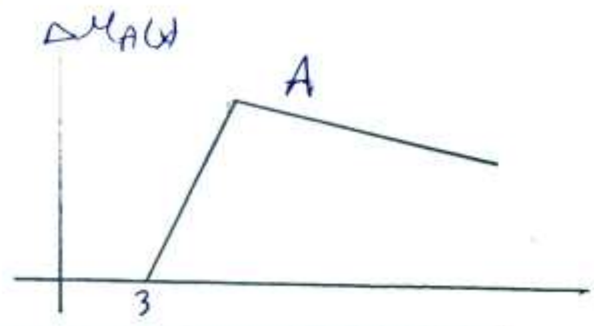
1) Support:-

support (A) =]a, d[



↳ the elements (members) of fuzzy set where its MF (membership function) degree $\neq 0$ $[\mu_A(x) \neq 0]$

support(A) =]3, ∞[

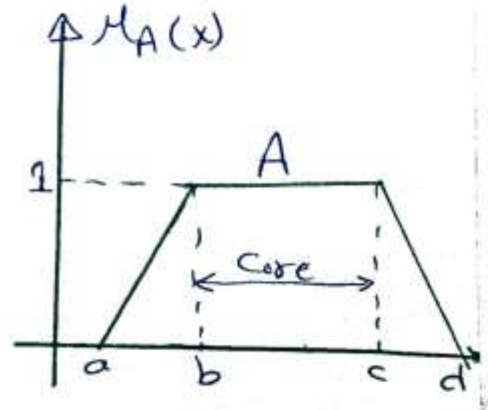


2 Core:-

core(A) = [b, c]

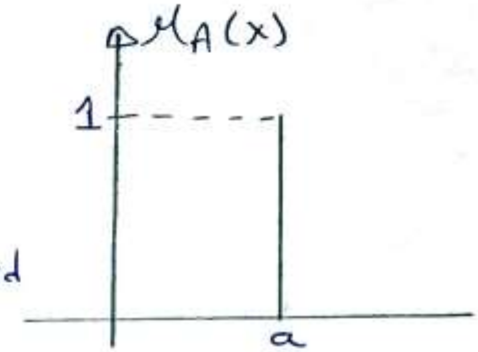
→ the elements of fuzzy set where its MF degree = 1

$(\mu_A(x) = 1)$



[3] Single ton

→ when the no. of elements of fuzzy set is equal to 1 with $\mu = 1$, the fuzzy set is called single ton fuzzy set.



"classification of fuzzy sets"

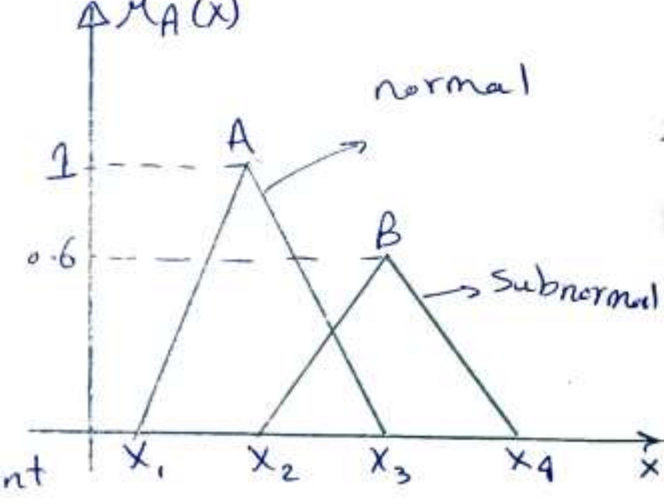
[1] Normal fuzzy sets & subnormal fuzzy sets :.

Normal fuzzy set :

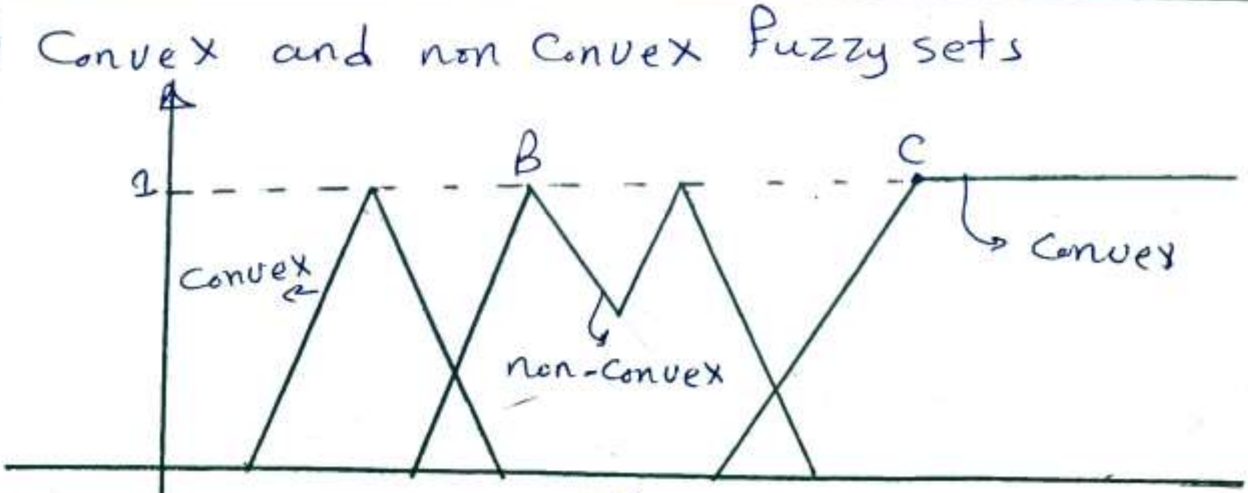
contains at least one element with $\mu_A = 1$

Subnormal fuzzy set

doesn't contain any element with $\mu = 1$



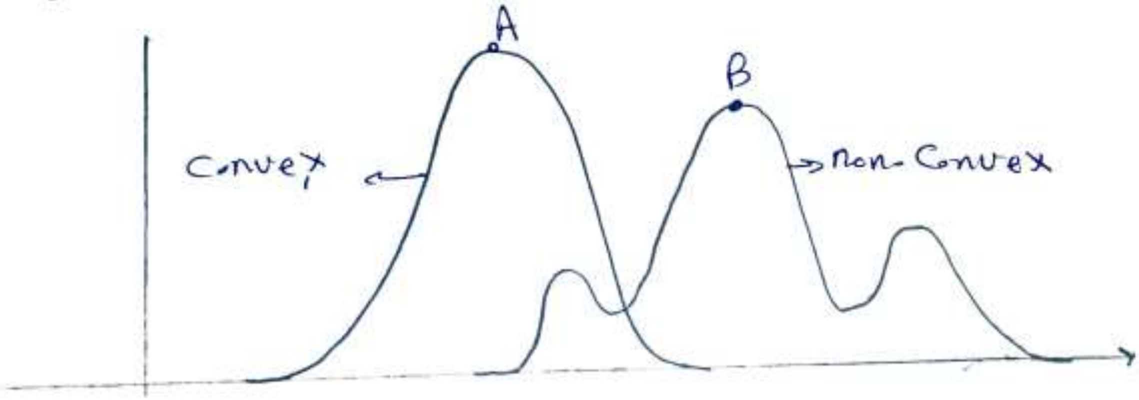
2) Convex and non convex fuzzy sets



ـ في الشكل السابق :

A ـ μ زادت من 0 ← 1 ولما قلت ، بتقل بشكل مرتب من 1 ← 0 .

B ـ زادت من 0 ← 1 ولكنها قلت وموصلتش للصفر .



* Convex fuzzy sets

↳ fuzzy set is convex when its μ is monotonically increasing or decreasing or increasing and decreasing over the elements of the set.

→ range must be from 0 → 1 to be normal fuzzy set.

* In design process of fuzzy controller, we need fuzzy sets to be:

1) Normal fuzzy sets.

2) Convex " " .

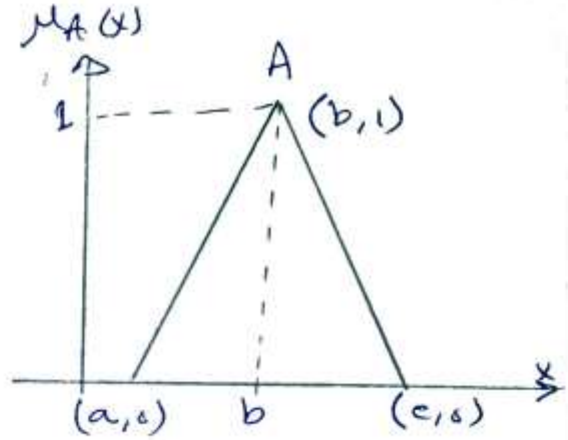
3) has bounded support.

"Common fuzzy sets"

[1] Triangular fuzzy set:-

← من أشهر الأنواع، سهل التعريف.

↳ has 3 tuning Parameters that control the shape of the fuzzy set.



$$a \le b \le c$$

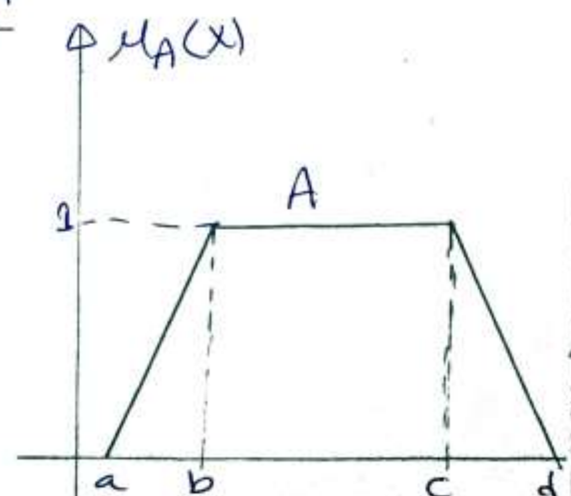
$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{x-c}{b-c} & b \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_A(x) = \max\left[\min\left(\frac{x-a}{b-a}, \frac{x-c}{b-c}\right), 0\right]$$

[2] Trapezoidal fuzzy set

↳ has 4 tunning Parameters that Control the fuzzy set.

$$a \le b \le c \le d$$



$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \dfrac{x-d}{c-d} & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_A(x) = \max\left[\min\left(\frac{x-a}{b-a}, 1, \frac{x-d}{c-d}\right), 0\right]$$

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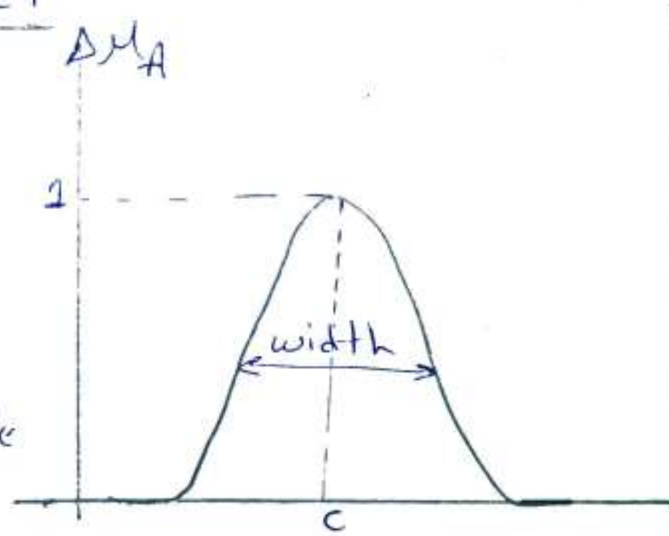
3 Gaussian fuzzy set

↳ has 2 tunning Parameters that control fuzzy set (c, w)

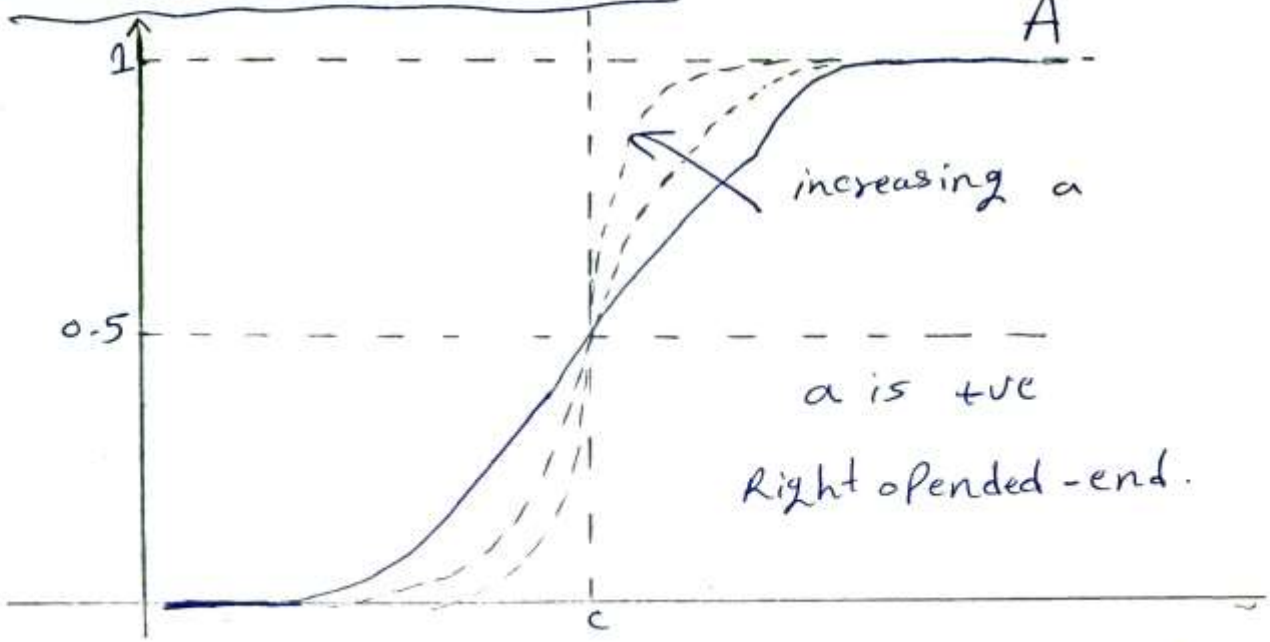
~~w → width of~~

$w \rightarrow$ control width of shape



$$\mu_A(x) = e^{-0.5\left(\frac{x-c}{w}\right)^2}$$

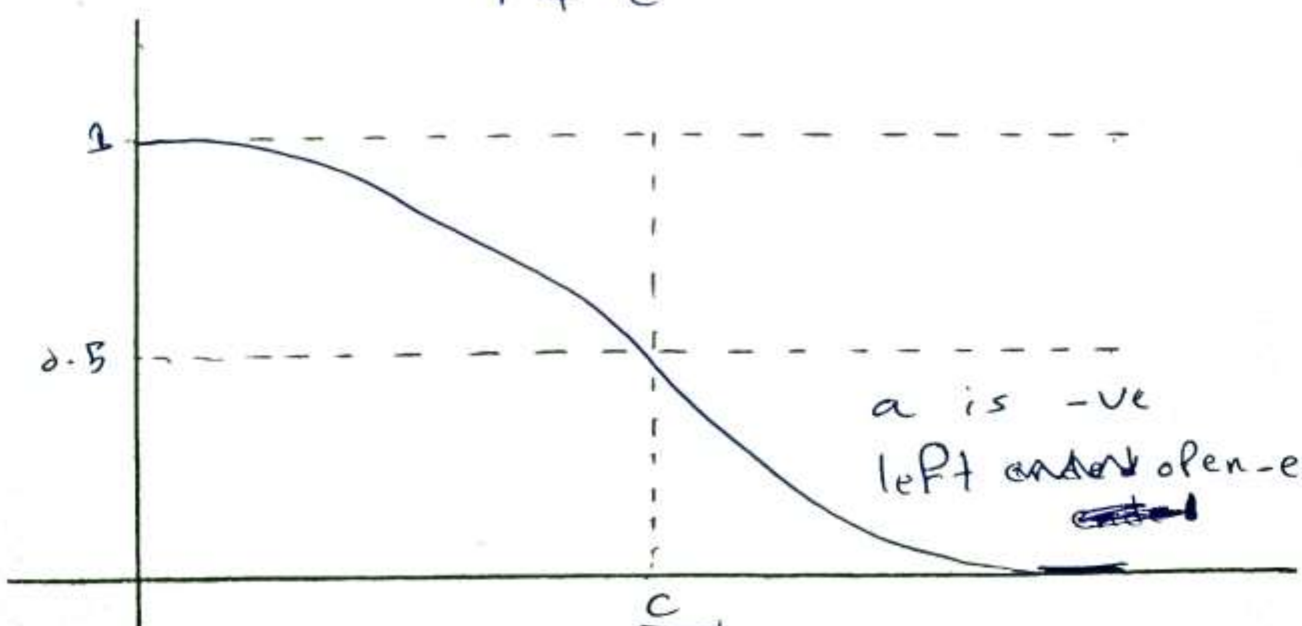
4 Sigmoidal fuzzy set



→ has 2 tuning parameters that control the shape of the fuzzy sets a, c.

→ a control the slope of this shape.

$$\mu_A(x) = \frac{1}{1 + e^{-a(x-c)}}$$



→ the sign of a determine the open-end of the shape
- → +ve (Right open end)
- → -ve (left open end)

← شكل الأوامر على الماتلاب :

- gaussmf(x, [w, c])
- trapmf(x, [a b c d])
- trimf (x, [a b c])
- sigmf (x, [a c])

"operations of fuzzy sets"

1) union operation (Connective or operator)

↳ union of two fuzzy sets A, B is denoted by : $C = A \cup B$

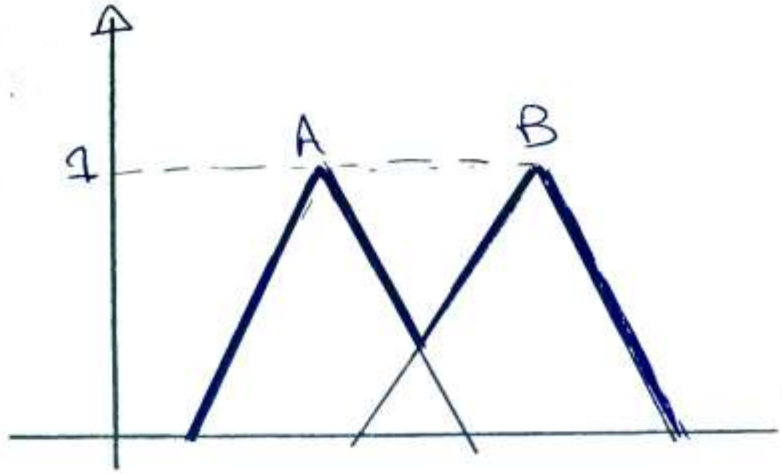
where:

$\mu_c(x)$:

→ Max-operation
$$\mu_c(x) = \max(\mu_A(x), \mu_B(x))$$

↳ Product operation
$$\mu_c(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

Ex



ـ الخط التقيل
ده الإتحاد
$A \cup B$

Ex → The fuzzy set A = "high temp." defined as:

$$A = \left\{ \frac{0}{36.5} + \frac{0}{37} + \frac{0.1}{37.5} + \frac{0.5}{38} + \frac{0.8}{38.5} + \frac{1}{39} + \frac{1}{39.5} + \frac{1}{40} \right\}$$

and the fuzzy set B = "Dangerous temp" defind as

$$B = \left\{ \frac{0}{37.5} , \frac{0.1}{38} + \frac{0.2}{38.5} + \frac{0.5}{39} + \frac{1}{40} + \frac{0.8}{39.5} \right\}$$

find $A \cup B$ = high or dang. temp

→ using max operation

$$C = A \cup B = \left\{ \frac{0}{36.5} + \frac{0}{37} + \frac{0.1}{37.5} + \frac{0.5}{38} + \frac{0.8}{38.5} + \frac{1}{39} + \frac{1}{39.5} + \frac{1}{40} \right\}$$

$C = A \cup B$ using Product operation

$$= \left\{ \frac{0}{36.5} + \frac{0}{37} + \frac{0.1}{37.5} + \frac{0.1}{37.5} + \frac{0.55}{38} + \dots \right\}$$

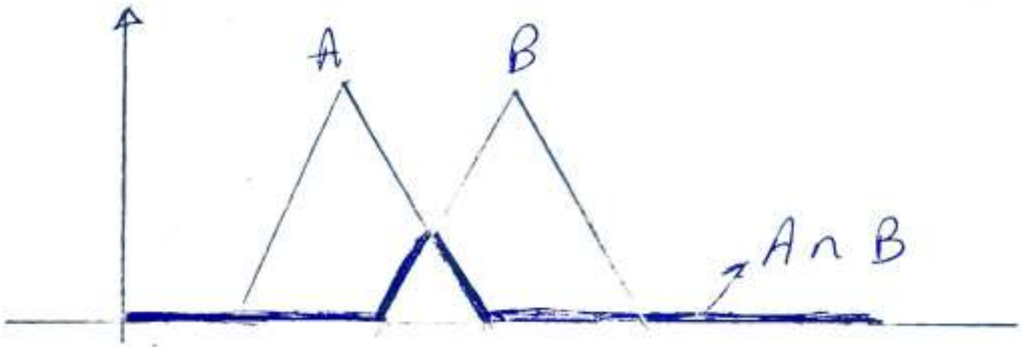
له ممكن يطلع أرقام مختلفة

$\boxed{2}$ Intersection operation (And Connective operator)

↳ the intersection of two fuzzy sets A and B is defined as:

$$C = A \cap B$$

where:
- → minimum rule ⇒ $\min(\mu_A(x), \mu_B(x))$
- ↳ Product rule ⇒ $\mu_C(x) = \mu_A(x) \cdot \mu_B(x)$



$\boxed{3}$ The complement operation (NOT operation)

~ complement of fuzzy set A is denoted as: $\bar{A}$ or $\sim A$ represent to what degree the element doesn't belong to fuzzy set.

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$